ELECTROMAGNETIC WAVE

Electromagnetic waves- Electromagnetic waves or EM waves are waves that are created as a result of vibration between an electric and magnetic field. In other words, EM waves are composed of oscillating magnetic and electric fields.

MAXWELL BASIC EQUATION.

As we know that equations governing static electric field due to charge at rest and static magnetic field due to steady current are already known. These equations are already known and given by Gauss, Ampere and Faraday.

i> From Gauss's Law

$$\nabla \cdot B = 0$$

$$\nabla \cdot D = \rho$$

ii> From Faraday's Law

$$\nabla \times E = \frac{-\partial B}{\partial t}$$

iii> From Ampere's Law

$$\nabla \times H = J$$

where
B = Magnetic flux density.
D = Electric field flux density.
E = Electric field intensity.
H = magnetic field intensity.
J = Current density.

Equation of continuity for steady current is given by

$$\nabla \cdot J = 0$$

Page No.:
Date: / /

Maxwell's Equation in differential form.

1) From Gauss's law

$$\nabla \cdot B = 0$$

$$\nabla \cdot D = \rho$$

2) From Faraday law

$$\nabla \times E = -\frac{\partial B}{\partial t} = -\dot{B}$$

3) From Ampere's circuital law

$$\nabla \times H = J + \frac{\partial D}{\partial t}$$

$$= J + \dot{D}$$

where

D = Electric displacement density in C/m²

B = Magnetic induction or Magnetic flux density in weber/

E = Electric field intensity or Electric field strength or simply electric field, in volts/m.

H = Magnetic field intensity, in Ampere/m

J = Conduction current density in Ampere/

ρ = Free charge density.

DISPLACEMENT CURRENT

The concept of displacement current was first introduced by Maxwell. He postulated that not only the displacement current produces magnetic field but also changing

This current produce the same magnetic field effect changing electric field in a vacuum or in a dielectric produces a magnetic field.
This current (displacement current) produces the same effect as an ordinary current in a conductor. This current is known as displacement current.

Consider a parallel plate capacitor connected through a switch 'S' and resistor R to the battery. Initially the switch is open, the plates of capacitor are uncharged and there is no net positive or negative charge exists on either plate. Now, when the switch is closed, electrons are drawn from upper plate to the positive terminal of the battery through R. This create a net positive charge on the top of the plate and -ve charge on the bottom down plate of capacitor.

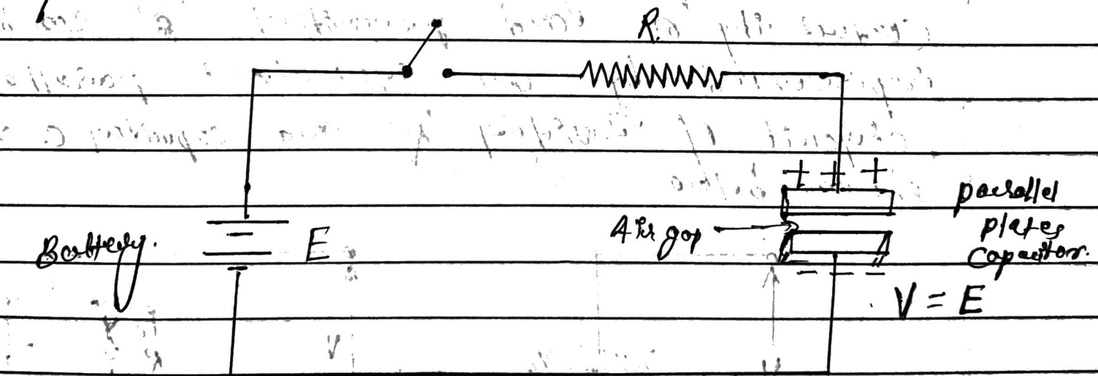


Fig : parallel plate capacitor.

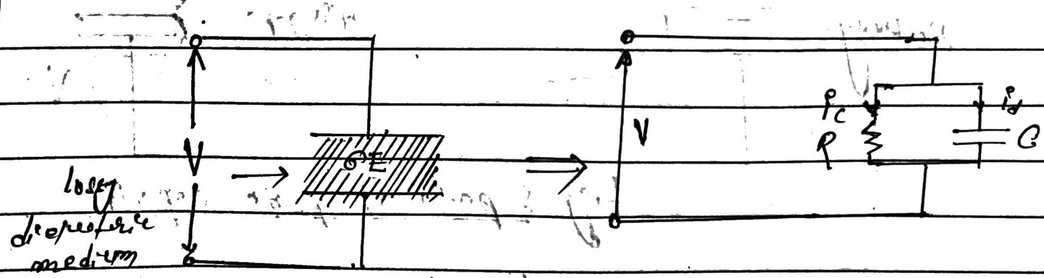
This transfer of electrons continues until the potential difference across the plate is equal to the battery potential.

Page No.:
Date : / /

During charging, there is no actual flow of charge between plates. Now, if we place a compass needle in space between the plates, the ~~middle~~ needle deflects. This indicates that there is a magnetic field between capacitor plates, even there is no flow of charge. This is because changing electric field between plates.

The electric field arises from moving charge and is varying with time, thus producing a magnetic field proportional to electric field. Displacement current associated with time varying electric fields. So displacement current is defined as the time rate of change of electric flux through the surface.

A capacitor of lossy dielectric material having conductivity 'σ' and permittivity 'ϵ' can be represented by an equivalent parallel circuit of resistor 'R' and capacitor C as shown below.



Let

V = time varying voltage

i_c = current through the resistor, R.

Page No.:
Date : / /

i_d = current through the capacitor, C.

Now,
The current through the resistor is due to actual motion of charge. This current is known as conduction current and is given by

$$i_c = \frac{V}{R} \quad \text{---} (I)$$

The current flowing the capacitor is displacement current (i_d)

$$i_d = \frac{dq}{dt} = C\frac{dV}{dt} \quad \text{---} (II)$$

Suppose, area of two plates is 'A' and separated by a distance 'd' with dielectric having permittivity ϵ, then

$$C = \frac{\epsilon A}{d}$$

On substituting the value of C in equation (II) we get

$$i_d = \frac{\epsilon A}{d} \cdot \frac{dV}{dt}$$

$\therefore \quad E = \frac{V}{d}, \quad \therefore V = Ed$

put $V = Ed$ in above equation we get

$$i_d = \frac{\epsilon A}{d} \frac{d\,E\cancel{d}}{dt}$$

$$= \epsilon A \frac{dE}{dt}$$

also,

$$\frac{i_d}{A} = \epsilon \frac{dE}{dt}$$

$$J_d = \epsilon \frac{dE}{dt} \qquad \left[\because J_d = \frac{i_d}{A} \right.$$

called displacement current density]

also, $J_d = \frac{d\epsilon E}{dt} = \frac{dD}{dt}$ [$\because D = \epsilon E$]

Since D is varying with space

$\therefore J_d = \frac{\partial D}{\partial t}$ — (III)

Now, If A is the area of cross-section of resistor, Then conduction current density is given by $J_c = \frac{I_c}{A} = \sigma E$

$\therefore$ Total current density (J) is given by

$J = J_c + J_d$

$$\boxed{J = \sigma E + \frac{\partial D}{\partial t}}$$

WAVE EQUATION FOR FREE-SPACE OR LOSSLESS OR NON-CONDUCTING MEDIUM.

Consider an ~~wave~~ electromagnetic wave propagating in free space or in non-conducting or loss-less or perfect dielectric medium in which there is no conduction current ($J = 0$) and no charge existed (i.e. $\rho = 0$) Then the Maxwells equation becomes

$$\nabla \times H = \frac{\partial D}{\partial t} = \epsilon \frac{\partial E}{\partial t} \quad \text{— (I)}$$

$$\nabla \times E = -\frac{\partial B}{\partial t} = -\mu \frac{\partial H}{\partial t} \quad \text{— (II)}$$

$$\nabla \cdot D = 0, \quad \nabla \cdot E = 0 \quad \text{— (III)}$$

$$\nabla \cdot B = 0, \quad \nabla \cdot H = 0 \quad \text{— (IV)}$$